and it was with great joy that our families got together every summer.

We share, with his wife Gaby and two daughters Dominique and Anne, the feeling of having lost an exceptional human being.

Notes

1. In fact Borel stayed only one year in Paris.
2. The conjecture indeed appears as a question in a letter (dated April 2nd, 1953) from Borel to Serre commenting on Mostow's result.

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A solution of the Ten Martini problem

The Ten Martini problem asks for the Cantor structure of the so-called Almost Mathieu or Harper operator which is the following operator

$$\left(H_{b,\omega,\phi}x\right)_n = qx_{n+1} + x_{n-1} + b\cos\left(2\pi\omega n + \phi\right) x_n.$$

on $l^2(\mathbb{Z})$ where b is a coupling parameter, ω is a frequency and ϕ a phase.

Apart from its naturality (it is probably the "simplest" example of a quasi-periodic Schrödinger operator) this operator appears in the study of the Hamiltonian of an electron in a rectangular lattice subject to a perpendicular magnetic field. The frequency ω stands for the intensity of the magnetic field while $b/2$ takes into account the nearest neighbour couplings. This model was introduced by Peierls and Harper in 1955 and Azbel, in 1962, conjectured that the spectrum of the operator for irrational frequencies (which does not depend on ϕ for such irrational values) should be a Cantor set if $b \neq 0$. This conjecture was strengthened by the numerical experiments of Hofstadter (1976) and Aubry (1977) who, in addition, conjectured that the measure of the spectrum should be zero in the "square case" $|b| = 2$. Such numerical computations are usually referred as "Hofstadter butterflies": the spectrum is plotted against the frequency and a fixed value of the coupling constant (see Figure 1).

In the beginning of the eighties there was a lot of work in the mathematical theory of almost periodic Schrödinger operators and, in fact, the name of the "Ten Martini problem" was coined by Simon after an offer by Kac in a meeting of the American Mathematical Society in 1981. The problem, which appeared in a famous list of problems in almost periodic Schrödinger operators (1982) remained open until recently.

Previous partial results include Bellissard & Simon (1982) who prove Cantor structure for generic pairs of (b,ω); Sinai (1987) proved that if ω satisfies a Diophantine condition then the spectrum is a Cantor set provided $|b|$ is small enough (how small depending on the precise Diophantine condition). These Diophantine frequencies, a total measure subset of the real numbers, are characterized by being "far from rational numbers". Regarding non-Diophantine irrational numbers, the so-called Liouville numbers, Choi, Elliot & Yui (1991) proved the Ten Martini Problem for a class of these Liouville numbers.

The "square case" is special because the spectrum is a Cantor set of zero measure, as a series of works by Helffer & Sjöstrand (1989), Last (1994) and Avila & Krikorian (2004) show. This case is also know as the "self-dual" case because it is invariant by Fourier transform, known as "Aubry duality" in this context. More generally, the Fourier transform can be used to show that the spectrum of the Almost Mathieu oper-

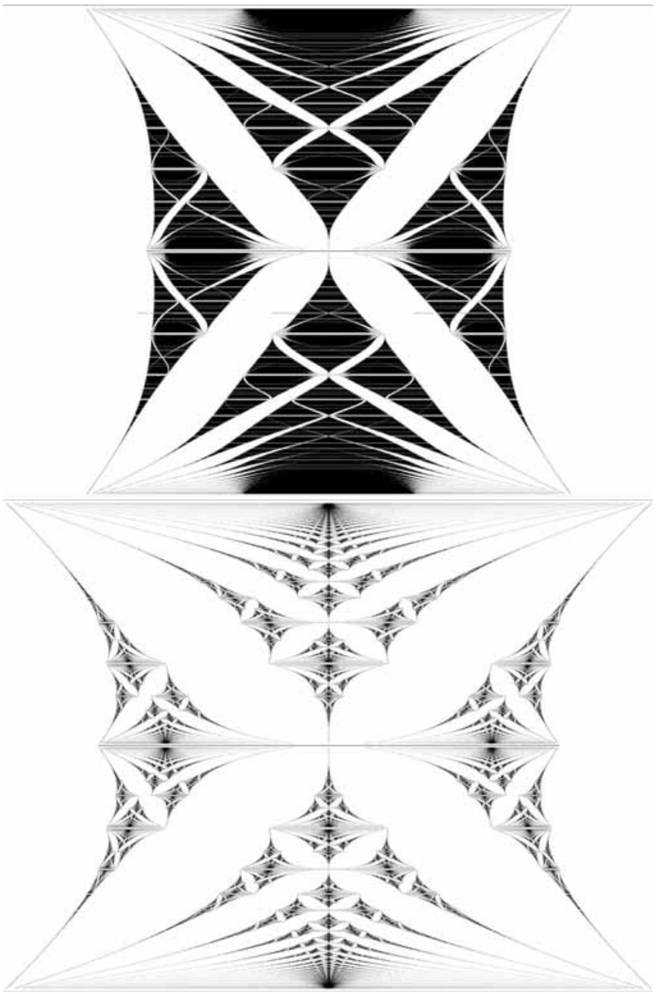

Figure 1. Hofstadter butterflies for values of $b = 1$ (top) and 2 (bottom) from top to bottom and left to right respectively. These pictures are computed taking rational frequencies ω in the vertical direction and computing the spectral bands of the periodic problem. The "classical" Hofstadter butterfly corresponds to the case $b = 2$ where the measure of the spectrum is zero for irrational frequencies.

ator for some $b \neq 0$ is just a dilatation of the spectrum for its "dual" value $2/b$.

Joaquim Puig, in the paper "Cantor spectrum for the Almost Mathieu operator" and as a part of his PhD thesis proved that if $|b| \neq 2$ and ω is Diophantine, then the spectrum of the Almost Mathieu operator is a Cantor set. The idea of the proof is the combination of tools coming from spectral theory and dynamical systems. On the spectral side a non-perturbative localization result due to Jitomirskaya (1999) is key for the proof. On the dynamical side, the concept of reducibility of quasi-periodic cocycles is also very important. While these two techniques can be generalized to other quasi-periodic Schrödinger operators, an adaption of Ince's (1922) argument for the classical Mathieu equation to the present almost periodic case is very specific of this model. The combination of these spectral and dynamical techniques for the study of quasi-periodic Schrödinger operators is powerful and opens a promising field of research. Recently, for instance, Avila & Jitomirskaya (preprint 2005) have obtained a proof of the Ten Martini problem which includes the non-Diophantine frequencies which were not covered by Choi, Elliot & Yui so that the Ten Martini problem is now settled for all irrational frequencies.

Joaquim Puig i Sadurní, presently at the Polytechnic University of Catalonia, received his PhD in Mathematics by the University of Barcelona with the thesis "Reducibility of Quasi-Periodic Skew-Products and the Spectrum of Schrödinger Operators", under the supervision of Carles Simó i Torres, on June 22nd 2004. Among the results in the thesis there is the proof of the conjecture known as the Ten Martini Problem for Diophantine frequencies.

Letter to the Editor: An Observation on Real Division Algebras

Holger P. Petersson, Hagen (Germany)

In [1] a short and elementary proof is given for the well-known fact that there are no associative real divison algebras of dimension 3. Without claiming originality, we present here an even shorter but still elementary proof for the following general observation that holds for both associative and non-associative division algebras:

Observation. *There are no real division algebras of odd dimension* > 1.

Proof. Suppose on the contrary that A is such an algebra and write $L_u : A \to A,\ v \mapsto L_u(v) = uv$, for the left multiplication by $u \in A$, which, thanks to our hypothesis, is a bijective linear transformation unless $u = 0$. Hence, given linearly independent vectors $x, y \in A$ and $t \in \mathbf{R},\ p(t) = \det L_{x+ty}$ is a real polynomial of odd degree without real roots, a contradiction. □

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