

Quantitative stochastic homogenization of convex energy functionals

I will describe recent results in stochastic homogenization for divergence-form, uniformly elliptic equations. Included are nonlinear equations arising as the first variation of uniformly convex energy functionals. The random field of coefficients is typically assumed to satisfy a finite range of dependence condition. Almost thirty years ago, Dal Maso and Modica proved a qualitative homogenization theorem covering this case. We are concerned with developing a quantitative theory-- that is, understanding the precise size and nature of the fluctuations of the solutions (and their energy density) from the homogenized limit. Here new ideas are needed, since the qualitative proof of Dal Maso and Modica was based on an abstract ergodic theorem which is not easily made quantitative. In joint work with Charles Smart, we introduce a method which leads to eventually to optimal quantitative estimates. Specializing to the case in which the Euler-Lagrange equation is linear, we get a new proof of recent results of Gloria, Neukamm and Otto as well as Marahrens and Otto. Some of the top-level ideas (although not the details) are parallel to those we have recently developed for non-divergence form equations, which I may also briefly review.