JISD 2012
Abstracts

Communications

Carlos Argáez (Dublin Institute of Technology)

"Elliptic Variational Problems with Nonlocal Operators"

Abstract. It is well-known that the use of relativistic calculations are mandatory if one is to obtain an accurate description of heavy atoms and ions. For this purpose we consider quasi-relativistic N-electron Coulomb systems describing heavy atoms; that is, systems where the kinetic energy of the electrons is given by the quasi-relativistic operator

$$\sqrt{-\alpha^{-2}\Delta + \alpha^{-4}} - \alpha^{-2},$$

where α is Sommerfeld's fine structure constant. This operator is a nonlocal, pseudo differential operator of order one. We present results on two different electronic structure models: (1) the Kohn-Sham model; (2) the multiconfigurative Hartree-Fock model. Both models give rise to locally compact variational problems with nonlocal operators.

(1) For spin-unpolarized systems in the local density approximation, we prove existence of a ground state (or minimizer) for the standard and extended Kohn-Sham model provided that the total charge Z_{tot} of K nuclei is greater than $N-1$ and that Z_{tot} is smaller than a critical charge $Z_{\text{c}} = 2\alpha^{-1}\pi^{-1}$. The proof is based on the concentration-compactness approach.

(2) We establish existence of infinitely many distinct solutions to the multi-configurative Hartree-Fock type equations. Finitely many of the solutions are interpreted as excited states of the molecule. Moreover, we prove existence of a ground state. The results are valid under the hypotheses that the total charge Z_{tot} of K nuclei is greater than $N-1$ and that Z_{tot} is smaller than a critical charge Z_{c}. The proofs are based on a new application of the Lions-Fang-Ghoussoub critical point approach to nonminimal solutions on a complete analytic Hilbert-Riemann manifold.

Begoña Barrios (Universidad Autónoma de Madrid)

"Bootstrap Regularity for Integro-differential Operators and its Application to Nonlocal Minimal Surfaces"

Motivated by the structure of interphases arising in phase transition models with long range interactions, in [CRS] the authors introduced a nonlocal version of minimal surfaces. A nonlocal minimal surface (NMS) is the boundary of a set whose characteristic function minimizes a fractional Sobolev norm $H^{s/2}$ for $s < 1$. That is, these objects minimizing a "nonlocal perimeter" inside a fixed domain. Recently has been proved that NMS approach the classical ones in the sense of the Γ-convergence and in a geometric point of view. Also it is proved in [CRS] that "flat s-minimal surface" are $C^{1,\alpha}$ for all $\alpha < s$. As far as we know all works related to NMS are only focused on the $C^{1,\alpha}$ regularity leaving the higher regularity as an open problem. In this talk we are going to explain how we can prove that in fact $C^{1,\alpha}$ nonlocal minimal surfaces are indeed C^∞. To obtain this result we will need to write the fractional minimal surface equations in a suitable form and establish, using the ideas developed in [CS], a very general result about the regularity of the integro-differential equations whose kernel is close to the ones of fractional Laplacian.

This work is joint whit Alessio Figalli (University of Texas at Austin) and Enrico Valdinoci (Università degli Studi di Milano).

Marta Canadell (Universitat de Barcelona)

Parameterization Methods for Computing Normally Hyperbolic Invariant Tori: some numerical examples

Marta Canadell. This is joint work with Alex Haro.

In this short communication I explain numerical algorithms for the computation of normally hyperbolic invariant tori. The algorithms are inspired in the parameterization method (Cabré, Fontich, de la Llave) for finding a parameterization of the invariant manifold and a dynamics on it. The framework leads to solving invariance equations, for which one uses a Newton method adapted to the dynamics and the geometry of the (invariant) manifold.

We present two algorithms:

- First algorithm computes the invariant torus and its unknown dynamics from an initial approximation of the torus and its invariant bundles. The algorithm is based on current work by A. Haro and R. de la Llave on the parameterization method for normally hyperbolic invariant manifolds.
- Second algorithm is based on a KAM scheme to find the parameterization of a torus with fixed Diophantine frequency (by adjusting parameters).

The algorithms are applied to continuation of saddle invariant curves, we consider a 3D-Fattened Arnold Family. With the first algorithm we continue the torus w.r.t. parameters regardless its dynamics, crossing resonances. With the second algorithm we continue a curve in the parameter plane for which the torus has a fixed frequency (Diophantine). We explore in both cases the mechanism of breakdown of the saddle invariant curve.

Jacek Cyranka (Jagiellonian University)

"Algorithms for Computer Assisted Proofs in PDEs"

We are dealing with nonlinear partial differential equations of dissipative type (dPDEs), we consider the following problem

$$\begin{aligned} u\colon [0,T) \times \mathbb{T} \to \mathbb{R}, \quad f, u_0 \colon \mathbb{T} \to \mathbb{R}, \\ u_t = L(u) + N(u, u_x, u_{xx}, \dots) + f, \\ u(0,x) = u_0(x). \end{aligned} \tag{1}$$

Where L is a "Laplacian" operator (for instance $L(u) = \nu u_{xx}$ or $L(u) = -\nu u_{xxxx} - u_{xx}$), N is a proper polynomial of u and its partial derivatives, f is a constant in time forcing, T is a maximal time of existence, $\mathbb{T}$ is the one-dimensional torus.

The Fourier basis is introduced and the problem of solving (1) is reduced to the problem of solving the following infinite dimensional dynamical system

$$\frac{da_k}{dt} = c_N k^r \sum_{\substack{k_1,\dots,k_n \in \mathbb{Z} \\ k_1 + \dots + k_n = k}} a_{k_1} \cdot a_{k_2} \cdots a_{k_n} + \lambda_k a_k + f_k, \quad k \in \mathbb{Z}. \tag{2}$$

Where λ_k are eigenvalues of the "Laplacian", c_N, n and r depends on N, $\{a_k\}_{k \in \mathbb{Z}} \subset \mathbb{C}$ describes the evolution of the Fourier modes.

Assuming that the initial condition is sufficiently regular the solution of (2) exists and is unique, moreover it is a solution of (1). The solution of (2) can be explicitly bounded using a computer algorithm involving the interval arithmetics (rigorous integration in time).

In our research we have focused on efficient computational methods for the problem of rigorously integrating (2). We will present what is the main computational difficulty in the problem of rigorously integrating (2), and how it is circumvented by combining the FFT algorithm + the automatic differentiation.

We will present what are possible applications of the algorithm, what particular equations can be considered and explicitly state what are opportunities and limitations of such approach.

We will focus on particular example of the viscous Burgers equation with forcing

$$u_t = \nu u_{xx} - u \cdot u_x + f$$

for which we have established existence of a unique fixed point which is globally attracting using a combination of analytical estimates with computer assisted method.

Serena Dipierro (SISSA)

"Concentration of Solutions for a Singularly Perturbed Elliptic PDE Problem in Non-smooth Domains"

We consider the following singularly perturbed equation

$$-\epsilon^2 \Delta u + u = u^p \quad \text{in } \Omega,$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain whose boundary has an $(n-2)$-dimensional smooth singularity. We study the problem both with Neumann and with mixed Dirichlet and Neumann boundary conditions. Assuming $1 < p < (n+2)/(n-2)$, we prove that, in both cases, concentration of solutions occurs at suitable points of the non smooth part of the boundary as the singular perturbation parameter tends to zero.

Diego Sebastian Ledesma (UNICAMP)

"Foliated Stochastic Calculus and Harmonic Measures"

In this article we present an intrinsic construction of foliated Brownian motion via stochastic calculus adapted to foliation. The stochastic approach together with a proposed foliated vector calculus provide a natural method to work on harmonic measures. Other results include a decomposition of the Laplacian in terms of the foliated and basic Laplacians, a characterization of totally invariant measures and a differential equation for the density of harmonic measures.

Jason Mireles James (Rutgers University)

"Parameterization of Invariant Manifolds with Rigorous Computer Assisted Error Bounds"

I will begin with a brief overview of the Parameterization Method for non-resonant invariant manifolds. This is a general method, developed by X. Cabre, R. de la Llave, and E. Fontich, for studying stable and unstable manifolds in both discrete and continuous time dynamical systems. Then I'll discuss numerical implementation of the method on the digital computer. Here the emphasis will be on obtaining a high order polynomial representation of the manifold with mathematically rigorous bounds on the truncation error. Time permitting I'll discuss how these computations facilitate computer assisted proof of the existence of connecting orbits and chaos, as well as extensions systems with parameters.

Benhamidouche Noureddine (University of M'sila)

“Dynamical profiles solutions for some PDEs”

Self-similar solutions play an important role in the development of the theory of nonlinear evolutions PDEs. Not only they provide exact solutions, but they often describe the asymptotic behavior at large times, or blow-up at some finite time.

In this talk, we present a new approach to study a general self-similar dynamics to some nonlinear PDEs. The approach which we will present it called "the traveling profiles method" (TPM).

Consider the following equation:

$$\frac{\partial u}{\partial t} = A_x u \tag{1.1}$$

where $A_x u$ is a linear or nonlinear differential operator.

Our approach is to find a general self similar exact solutions (called traveling profiles), in the form:

$$u(x,t) = c(t)\,\psi\left(\frac{x-b(t)}{a(t)}\right), \text{ with } a,b,c \in R \tag{1.2}$$

where ψ is in L^2, that one will call "the based-profile", and the parameters $c(t), a(t), b(t)$ are real valued functions of t.

The coefficients $c(t), a(t), b(t)$ are determined by the solution of minimizition problem :

$$\min_{\dot{a},\dot{b},\dot{c}} \int_{-\infty}^{+\infty} \left|\frac{\partial u}{\partial t} - A_x u\right|^2 dx, \tag{1.3}$$

and the " based-profile" is determined as solution of a differential equation.

Anupam Priyadarshi (Indian Institute of Technoloy Roorkee)

“Role of Protection in tri-trophic food chain dynamics”

In my talk, I shall discuss the role of protection in stabilizing the tri-trophic food chain dynamics. My model consists of three species, where, the density-dependent protection is provided to bottom prey or middle predator or both. The bifurcation diagrams have been drawn with respect to protection parameter, which shows the coexistence of all three species in the form of periodic solutions. The coexistence in the form of stable equilibrium is also possible for higher values of protection parameters. Further increase in protection parameters may lead to extinction of one or two species. The protection favors the oscillations damping and has the potential to control the chaotic fluctuations of population density. The Poincar´e Maps further confirm the role of protection in controlling the chaos.

Marco Sansottera (University of Namur)

"On the Secular Evolution of Extrasolar Planetary Systems"

In 1995, Mayor and Queloz announced the first detection of an extrasolar planet around a main-sequence star and nowadays more than 700 have been detected. One of the major issues concerns the uncertainties in the determination of the orbital parameters. For this reason, in order to investigate the dynamics, some open domains around the initial data deserve to be studied in detail. In this sense analytical tools, such as normal forms, are suitable.

Considering the exoplanetary systems, results on the secular evolution and the proximity to a mean motion resonance have been obtained considering a secular Hamiltonian at first order in the masses (see, e.g., [1] and [2]). This reliable analytical model can take into account a large range of orbital parameters (including large eccentricities of the planets) and reproduces successfully the dynamics of non-resonant systems. On the other hand, the study of the long time stability of the main bodies of the Solar System, improving the classical circular approximation by replacing it with a torus which is invariant up to order two in the masses, has given interesting results concerning realistic estimates. In particular, the stability of the Sun-Jupiter-Saturn around a KAM torus (see, e.g., [3]) and the stability of the secular evolution of the planar Sun-Jupiter-Saturn-Uranus system (see, e.g., [4]) have been investigated.

In the present work, we study the secular dynamics of exoplanetary systems consisting of two coplanar planets, considering a Hamiltonian at second order in the masses, to estimate the benefits of a second order model on the study of their long-term evolution. An additional aim of this work is to study the effects of the proximity to a mean motion resonance on the secular behavior of the planets. In the following we give a description of the main aspects.

In order to obtain a good description of the secular dynamics, a straightforward method is to include in the unperturbed Hamiltonian also the average of the perturbation over the fast angles; this is the so-called approximation at order one in the masses (see, e.g., [1]). However, if the system is near a mean motion resonance, the frequencies of the quasi-periodic flow given by this secular Hamiltonian are quite different from the true ones. The reason lies in the effect of the mean motion quasi-resonance. Therefore we look for an approximation of the secular Hamiltonian up to order two in the masses. To this end we perform a *Kolmogorov-like* normalization step in order to eliminate the main perturbation terms depending on the fast angles. Our aim is to replace the orbit with zero eccentricity with a quasi-periodic one that takes into account the effects of such resonance up to the second order in the masses.

The approximation of lowest order in the eccentricities of the secular Hamiltonian, namely its quadratic part, is essentially the one considered in the Lagrange-Laplace theory. In modern language, we say that the origin of the secular phase space is an elliptic equilibrium point. Following a quite standard procedure, we construct a high order Birkhoff normal form for the Hamiltonian using the Lie series method.

Considering the Hamiltonian in non-resonant Birkhoff normal form, the equations of motion take a very simple form, being function of the actions only. So, using the secular frequencies that are easily computed, the long term motion of the planets can be easily integrated analytically. By comparing the semi-analytical results based on the secular approximation, with the dynamics of the complete system, we can better understand whether resonant contributions dominate the evolution of the planets or not.

To conclude, let us remark the main difficulty that we must face, in addition to the usual *small divisors problem*, when we consider systems in the proximity to a mean motion resonance in the extrasolar planetary systems: the eccentricities of the planets are considerably larger than the ones of our Solar System. In spite of this, our expansion of the Hamiltonian is able to describe with a great accuracy many of the exoplanetary systems discovered so far.

Jirina Vodova (Mathematical Institute in Opava, Silesian University in Opava)

"Low-Order Hamiltonian Operators Having Momentum"

We describe all fifth-order Hamiltonian operators in one dependent and one independent variable that possess the momentum, i.e., for which there exists a Hamiltonian associated with translation in the independent variable. Similar results for first- and third- order Hamiltonian operators were obtained earlier by Mokhov. For further details please see the preprint arXiv:1111.6434.

Barbora Volna (Mathematical Institute in Opava, Silesian University in Opava)

"Relaxation Oscillations Emerging in Macroeconomics"

In this paper, we prove the existence of relaxation oscillations in one of fundamental macroeconomic model called IS-LM model. This model explains aggregate macroeconomic equilibrium - the equilibrium on goods market and money market simultaneously. We create new version of IS-LM model which eliminates two main deficiencies of original model. This model is continuous dynamical system. In these days, many experts and also the public more and more talk about unexpected fluctuation of different phenomenons in economics and about impact of these fluctuations on economics. This new IS-LM model with relaxation oscillations can explain some unexpected fluctuation of aggregate income on goods market and of interest rate on money or financial assets market.

Posters

Silvia-Otilia Corduneanu (University of Iasi):

"Differential equations involving the mean of almost periodic functions"

Nancy González (Universidad Nacional Autónoma de México):

"Non Local Bifurcations"

Dhananjay Gopal (SV National Institute of Technology):

"Absorbing maps and common fixed points"

Martin Himmel (University of Mainz):

"Singular behaviour of some divergence type PDE"

Oswaldo Larreal (Universidad del Zulia):

"An Approach To The Border Of The Acceptance"

Isamu Ohnishi (Hiroshima University):

"Bifurcation analysis to Legiato-Lefever equation in one space dimension"

Elżbieta Puźniakowska-Gałuch (University of Gdansk):

"Initial problems for hyperbolic functional differential systems"

Diana Stan (ICMAT and Universidad Autonoma de Madrid):

"Asymptotic behaviour of the doubly nonlinear equation in the degenerate and quasilinear cases"

Miguel Yangari (University of Chile):

"Front propagation for fractional KPP equations with slowly decaying initial conditions"