Ninth
JORNADES D'INTERACCIÓ ENTRE SISTEMES DINÀMICS I EQUACIONS EN DERIVADES PARCIALS (JISD2011)

Barcelona, June 27-July 01, 2011

The ninth edition of the JORNADES D'INTERACCIÓ ENTRE SISTEMES DINÀMICS I EQUACIONS EN DERIVADES PARCIALS (JISD2011) will be held in Barcelona, June 27-July 01, 2011 at the **Universitat Politècnica de Catalunya (UPC)**.

Past Editions of the JISD: 2002 - 2003 - 2004 - 2005 - 2006 - 2007 - 2008 - 2010

There will be four main courses, taught by Luis Caffarelli, Lorenzo Díaz, Vadim Kaloshin, and Yanyan Li, within the Master of Science in Advanced Mathematics and Mathematical Engineering (MAMME) of the UPC Graduate School. There will be also communications and posters by the participants.

Supported by the FME, UPC, SCM, I-Math.

Organizers
- Xavier Cabré
- Maria del Mar González
- Tere M. Seara

Scientific Committee
- Massimiliano Berti (Univ. Federico II)
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- Jean-Michel Roquejoffre (Univ. Paul Sabatier. Toulouse)
- Alfonso Sorrentino (Univ. Of Cambridge)
- Marco Antonio Teixeira (Univ. Estatal de Campinas)
- Juan Luis Vázquez (UAM)

There will be some ***financial support*** available for this edition. Deadline to apply for financial support: April 15, 2011.

ACCOMMODATION: suggested hotels

ninthJISD'2011

See the courses' **schedule** *here*

Contents

Courses will be held in the room 002 of the FME building (Facultat de Matemàtiques i Estadística), at C/ Pau Gargallo, n. 5 Barcelona, 08028.

Course	Abstract
Fully non linear equations I: Basic theory for local and non local elliptic equations *Luis Caffarelli* (Univ. of Texas Austin, USA) (Syllabus)	In the first half of the course we will present the main ideas of the regularity theory for solutions of fully non linear equations (sups and inf sups of linear operators) both in the second order and non local (integral) case: Alexandrov Bakelman Pucci, Krylov Safanov, Evans Krylov.
Fully non linear equations	In the second half of the course we will introduce a fully nonlinear version of the Yamabe

II: Applications to conformal geometry *Yanyan Li* (Rutgers University, USA) (Syllabus)	problem, and will describe results on the existence and compactness of solutions, associated Liouville theorems, comparison theorems, derivative estimates, for solutions of conformally invariant second order fully nonlinear elliptic and degenerate elliptic equations. We will also present, for general fully nonlinear elliptic (but not necessarily uniformly elliptic) equations, results on removable singularities, the strong maximum principle and Hopf Lemma for viscosity solutions.
Rich dynamics in heterodimensional cycles *Lorenzo Díaz* (Catholic University of Rio de Janeiro) (Syllabus)	We will consider several dynamical settings having a "rich dynamics" associated to the generation and unfolding of heterodimensional cycles (i.e., cycles involving two or more saddles whose stable directions have different dimensions). Heterodimensional cycles generates a sort of dynamical plug we call a blender. The dynamical effect of such blenders is that several hyperbolic sets of different stable dimensions are intermingled and contained in a bigger (non-hyperbolic) transitive set. Moreover, this dynamical configuration persists under small perturbations. We will systematically explore these facts. Our main focus will be C^1 dynamics.
Principle of minimal action in dynamics *Vadim Kaloshin* (University of Maryland) (Syllabus)	The motion of classical mechanical systems is determined by Hamiltonian differential equations. If Hamiltonian is convex in momentum, then its equations are equivalent to the Euler-Lagrange equations. Solutions to the Euler-Lagrange equation extremize action with fixed end points on each finite time interval. An important class of extremizers are minimizers. We investigate the minimal action in four di?erent settings:
Seminar	
Convolution type estimates for the Boltzmann collision operator in the regularity of the Boltzmann equation *Irene Gamba* (Univ. of Texas Austin, USA)	The Boltzmann integral has a weighted convolutional structure that, depending on the collisional kernels yields either Young's inequality for hard potentials or Hardy Littlewood Sobolev inequality for soft ones. These estimates are developed and shown to be a powerful tool to prove a gain of integrability that imply the propagation of regularity for the space homogeneous problem and also the propagation of L^p regularity and stability for the space inhomogeneous problem for soft potentials with initial data near local Maxwellian states. This is work in collaboration with Ricardo Alonso.
Some remarks on solvability of elliptic problem involving the Hardy potential *Ireneo Peral* (Univ. Autónoma de Madrid)	We analyze some semi-linear elliptic problems involving the Hardy potential $-\Delta u = f(\lambda/\|x\|^2, u, \nabla u)$ in a bounded domain Ω and with convenient hypotheses on f. We will focus the attention in the role that plays the position of the pole in $\overline\Omega$. These results are joint work with J.D. Dávila from Universidad de Chile, Santiago (Chile), and Susana Merchan from U.A.M. respectively.

(*) For further details, please contact Xavier.Cabre upc.edu, Mar.Gonzalez upc.edu, or Tere.M-Seara upc.edu

22-6-2011 - RosaMariaCuevas

Accommodation

(mention always that you come to a course at the UPC, because we have special prices):

There are several student residences in Barcelona, which offer very good prices.
Contact: resa@resa.es
Web: http://www.resa.es/eng/

If you wish to reserve a room in another hotel we suggest the following (all of them are located in the city center):

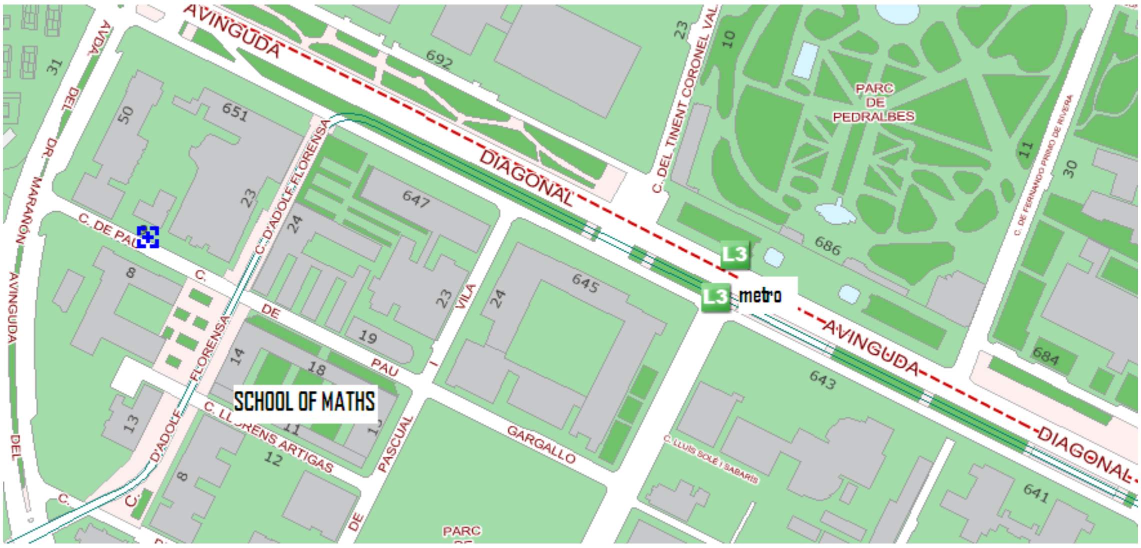
Hotel Catalonia Ramblas 4*
Single room: 120 € (taxes and breakfast included)
Double room 140 € (taxes and breakfast included)

Hotel Catalonia Plaza Cataluña 4*
Single room: 102 € (taxes and breakfast included)
Double room 120 € (taxes and breakfast included)

Hotel Catalonia Portal de l'Àngel 3*
Hotel Catalonia Port 3*
Hotel Catalonia Avinyó 3*
Single room: 89 € (taxes and breakfast included)
Double room 99 € (taxes and breakfast included)

Hotel Turin
Single room: 86,67 (taxes and breakfast included)
Double room: 130,97 (taxes and breakfast included)

To come to the school of maths from the city center you must take the underground, line 3, at Plaça de Catalunya or Liceu, and stop at Palau Reial. Here is a map that shows how to go to the School of Mathematics from the underground stop Palau Reial:



JISD2011 SCHEDULE

Monday June 27	08.00 - 09.00	**Welcome**
	09.00 - 10.30	**L. Caffarelli**
	10.30 - 11.00	Coffee break
	11.00 - 12.30	**L. Díaz**
	12.30 - 13.30	**L. Díaz**
	13.30 - 15.00	LUNCH
	15.00 - 16.00	**Y. Li**
	16.00 - 17.00	**Communications**

Tuesday June 28	09.00 - 10.30	**L. Díaz**
	10.30 - 11.00	Coffee break
	11.00 - 12.30	**L. Díaz**
	12.30 - 13.30	**Y. Li**
	13.30 - 15.00	LUNCH
	15.00 - 16.00	**L. Caffarelli**
	16.00 - 17.00	**Seminar: Irene Gamba**
	17.00 -	**POSTERS** - Social Gathering-

Wednesday June 29	09.00 - 10.30	**Y. Li**
	10.30 - 11.00	Coffee break
	11.00 - 12.30	**L. Díaz**
	12.30 - 13.30	**L. Caffarelli**
	13.30 - 15.00	LUNCH
	15.00 - 16.00	**V. Kaloshin**
	16.00 - 17.00	**Communications**

Thursday June 30	09.00 - 10.30	**Y. Li**
	10.30 - 11.00	Coffee break
	11.00 - 12.30	**V. Kaloshin**
	12.30 - 13.30	**Seminar: Ireneo Peral**
	13.30 - 15.00	LUNCH
	15.00 - 16.00	**V. Kaloshin**
	16.00 - 17.00	**Communications**

Friday July 1	09.00 - 10.30	**V. Kaloshin**
	10.30 - 11.00	Coffee break
	11.00 - 12.30	**L. Caffarelli**
	12.30 - 13.30	**V. Kaloshin**
	13.30 - 15.00	LUNCH

Click here to see the communications

For more information, please contact the organizers Xavier.Cabre□upc.edu, Mar.Gonzalez□upc.edu or Tere.M-Seara□upc.edu

June, 28-2011 - RosaMariaCuevas

Ninth
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Barcelona, June 27-July 01, 2011

Course	Syllabus
Fully non linear equations I: Basic theory for local and non local elliptic equations ***Luis Caffarelli*** (Univ. of Texas Austin, USA) ***Schedule***	• Fully non linear equations. Sups and inf sups of linear operators. Second order and non local (or integral) cases. • Alexandrov-Bakelman-Pucci type estimates • Krylov-Safanov type Harnack inequalities • Regularity theory of Evans-Krylov type
Fully non linear equations II: Applications to conformal geometry ***Yanyan Li*** (Rutgers University, USA) ***Schedule***	• Fully nonlinear version of the Yamabe problem. Conformally invariant second order fully nonlinear elliptic and degenerate elliptic equations • Existence and compactness of solutions, associated Liouville theorems, comparison theorems, derivative estimates. • Non-uniformly elliptic fully nonlinear elliptic equations: removable singularities, strong maximum principle and Hopf Lemma for viscosity solutions
Rich dynamics in heterodimensional cycles ***Lorenzo Díaz*** (Catholic University of Rio de Janeiro) ***Schedule***	• Description of blenders. Blenders are just transitive sets such that convenient projections of its stabl/unstable set have larger dimension than the stable/unstable dimension of the bundle. In some sense these sets play the role of the tick hyperbolic sets introduced by Newhouse, [2, 10, 11]. • Robust transitivity and minimality of foliations. We explain the construction of several examples of robsutly transitive diffeomorphisms and sets using blenders. We also analyze the role of the strong unstable and stable foliations, see [2, 9, 22, 23]. • Robust cycles. Following the pioneering work of Newhouse [19, 20, 21] about robust homoclinic tangencies of C^2 diffeomorphisms we see how heterodimensional cycles yield robust cycles and homoclinic tangencies, [1, 5, 6]. In the case of heterodimensional cycles this means the existence of two hyperbolic sets of different indices Λ and Σ of a diffeomorphism f such that there is an open set ${\cal U}$ close to f such that for every g in ${\cal U}$ the continautions Λ_g and Σ_g of Λ and Σ are involved in a cycle, i.e, $W^s(\Lambda_g) \cap W^u(\Sigma_g)\not= \emptyset$ and $W^u(\Lambda_g)\cap W^s(\Sigma_g)\not= \emptyset$. We also discuss the stabilization of heterodimensional cycles. • Wild dynamics We will discuss the generation of the so-called wild dynamics, meaning diffeomorphisms having infinitely many elementary pieces of dynamics (homoclinic classes, chain recurrence classes, sinks, repellors...) with some persistence. The first examples of this dynamics are the diffeomorphisms with infinitely many sinks associated to homoclinic tangencies, [20, 21], see also [16] We perform constructions in the same spirit whose ingredients are robust cycles and non-domination, [3, 8, 4]. • Dynamics of Homoclinic classes. Non-hyperbolic ergodic measures. The starting point is the ideas in [14] for constructing ergodic measures with zero Lyapunov exponents in

skew products. We see how this method togheter heterodimensional cycles can be used to construct ergodic measures with some zero Lyapunov exponent in the differentiable setting and with some persistence, [13, 7, 17, 18].

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Principle of minimal action in dynamics

Vadim Kaloshin (University

- Convex billiards
- Fixed points and invariant tori
- Hofer's geometry
- Symplectic geometry.

of Maryland) ***Schedule***	The book of Siburg ``Principal of minimal action in geometry and dynamics'' will be used.

9-3--2011 - RMC

Ninth JORNADES D'INTERACCIÓ ENTRE SISTEMES DINÀMICS I EQUACIONS EN DERIVADES PARCIALS (JISD2011)

Communications

Monday, June 27th, 4pm:

- Alberto Enciso (CSIC): *Knots and links in fluid mechanics.*
- Zhaosheng Feng (University of Texas-Pan American): *Bifurcation and proper solution of a parabolic system.*
- *Marc Mazade (University of Montpellier 2): Regularization of differential variational inequalities with locally prox-regular sets.*
- *Martijn Zaal (VU University Amsterdam): Modelling the swelling of a cell due to osmosis*

Wednesday, June 29th, 4pm:

- Agnese Di Casttro (University of Coimbra): *Limits of anisotropic and degenerate elliptic problems.*
- Fernando Charro (University of Texas at Austin): *The Aleksandrov-Bakelman-Pucci Maximum Principle revisited. Applications.*
- Dominika Pilarczyk (Instytut Matematyczny Uniwersytet Wrocławski): *Asymptotic stability of Landau solutions to Navier-Stokes system.*

Thursday, June 30th, 4pm:

- Hichem Chtioui (University of Sfax): *Existence results for the prescribed Scalar curvature on S^3.*
- Analia Silva (Universidad de Buenos Aires): *The concentration-compactness principle for variable exponent spaces and applications.*
- Dario Monticelli (Università degli Studi di Milano): *On fully nonlinear CR invariant equations on the Heisenberg group.*

Limits of anisotropic and degenerate elliptic problems

Agnese Di Castro,

CMUC, Centre for Mathematics, University of Coimbra

Abstract

Let Ω be a bounded, smooth and convex domain in $\mathbb{R}^N$, $f \in C(\overline{\Omega})$ a given function and consider the problem

$$\begin{cases} -\displaystyle\sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left(\left|\frac{\partial u}{\partial x_i}\right|^{p_i-2}\frac{\partial u}{\partial x_i}\right) = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where the exponents satisfy the condition $p_i > N$, for all $i = 1, \dots, N$. We analyze the behavior of solutions of this problem as the exponents go to infinity. We show that the solutions converge uniformly to a function that solves, in the viscosity sense, a certain problem that we identify. The results are presented in two-dimensional setting but can be extended to any dimension.

The Aleksandrov-Bakelman-Pucci Maximum Principle revisited. Applications

Fernando Charro

Abstract

In this talk we will prove the Aleksandrov-Bakelman-Pucci estimate for a class of (possibly degenerate) nonlinear elliptic and parabolic equations of the form

$$-\mathrm{div}\,(F\,(\nabla u(x))) = f\,(x) \qquad \text{in } \Omega \subset \mathbb{R}^n$$

and

$$u_t(x,t) - \mathrm{div}\,(F\,(\nabla u(x,t))) = f\,(x,t) \qquad \text{in } Q \subset \mathbb{R}^{n+1}$$

with F a C^1 monotone field under some suitable conditions. Examples of application such as the p-Laplacian or the Mean Curvature Flow are considered, as well as extensions of the general results to equations that are not in divergence form. This is a joint work with Roberto Argiolas and Ireneo Peral.

Existence results for the prescribed Scalar curvature on S^3

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November 14, 2009

Abstract

This paper is devoted to the existence of conformal metrics on S^3 with prescribed scalar curvature. We extend well known existence criteria due to Bahri-Coron.

1991 Mathematics Subject Classification. 12345, 54321.

Key words. Scalar curvature, critical points at infinity, topological method.

1 Introduction and the main result

On the sphere S^3 endowed with its standard metric g_0, a well studied question is the following one:
Given a function $K \in C^2(S^3)$, does there exist a metric g, conformally equivalent to the standard one whose scalar curvature is given by K? This a mounts to solve the following nonlinear PDE

$$\begin{cases} -Lu &= Ku^5 \\ \quad u &> 0 \qquad \text{on } S^3. \end{cases} \tag{1.1}$$

Where $-Lu = -8\Delta u + 6u$ is the conformal Laplacian operator of (S^3, g_0).
For the last four decades, scalar curvature problem has been continuing to be one of major subjects in nonlinear elliptic PDEs.

Unfortunately equation (1.1) does not have always a solution, indeed one can easily notice that a necessary condition is the function K is positive somewhere. Another deeper necessary condition is the so-called Kazdan-Warner Obstruction.
A sufficient condition was found by A. Bahri and J. M. Coron, through the theory of critical points at infinity it is an Euler-Hopf type criterium, namely they prove.

Theorem 1.1 *Under the following condition:*
(H_0) $0 < K \in C^2(S^3)$, *having only nondegenerate critical points such that*

$$\Delta K(y) \neq 0 \text{ for each } y, \text{ critical point of } K.$$

If

$$\sum_{y \in \mathcal{K}^+} (-1)^{3-ind(K,y)} \neq 1,$$

then (1.1) *has at least one solution.*
Where $\mathcal{K}^+ = \{y,\ \nabla K(y) = 0 \text{ and } -\Delta K(y) > 0\}$ *and* $ind(K,y)$ *denote the Morse index of* K *at* y.

A natural question which arises when looking to the above result, is what happens if the total sum is equal to 1, but a partial one is not. Under which condition can one use this partial sum to derive an existence result?
In order to give a partial answer to this question, we introduce the following condition:
We say that an integer $k \in (A_1)$ if it satisfies the following
(A_1) For each $z \in \mathcal{K}^+$, such that $3 - \text{ind}(K, z) = k + 1$ and for each $y \in \mathcal{K}^+$ such that $3 - \text{ind}(K, y) \leq k$, we have:

$$\frac{1}{K(z)^{\frac{1}{2}}} > \frac{1}{K(y_0)^{\frac{1}{2}}} + \frac{1}{K(y)^{\frac{1}{2}}},$$

where y_0 is an absolute maximum of the function K on S^3.
We are now ready to state our main result.

Theorem 1.2 *Let* $K \in C^2(S^3)$ *satisfying* (H_0), *if*

$$\underset{k \in (A_1)}{Max} \left| 1 - \sum_{\substack{y \in \mathcal{K}^+ \\ 3 - ind(K,y) \leq k}} (-1)^{3-ind(K,y)} \right| \neq 0$$

then there exist a solution to problem (1.1).

We observe that every $k \geq 2$, satisfies condition (A_1) since for every $y \in \mathcal{K}^+$, we have $\text{ind}(K, y) \in \{1, 2, 3\}$. It follows that the above mentioned celebrated result of Bahri-Coron is a corollary of our Theorem.

Alberto enciso (CSIC)

Title: Knots and links in fluid mechanics.

Abstract: In this talk we will consider the problem of the existence of knotted and linked trajectories in steady fluids. More precisely, we will prove that, given any locally finite link, we can transform it by a small diffeomorphism so that its transformed image is a set of periodic stream lines (or vorticity lines) of a steady solution of the incompressible Euler equation in R^3. This is based on joint work with D. Peralta-Salas (Ann. of Math., to appear).

Bifurcation and proper solution of a parabolic system

Zhaosheng Feng

Department of Mathematics, University of Texas-Pan American, Edinburg, TX 78539, USA

In this talk, we are concerned with a parabolic system and the associated ordinary differential systems. By applying the bifurcation theory and the Lie symmetry method, we find two nontrivial infinitesimal generators, and use them to construct canonical variables. Some dynamical properties of the nonlinear system under the certain parametric conditions are presented. Comparison with the existing results by the Prelle--Singer Procedure etc is provided. Under the same parametric conditions, properties of proper solutions are analyzed accordingly.

Regularization of differential variational inequalities with locally prox-regular sets

Marc Mazade and Lionel Thibault

Abstract

This paper studies, for a differential variational inequality involving a locally prox-regular set, a regularization process with a family of classical differential equations whose solutions converge to the solution of the differential variational inequality. The concept of local prox-regularity will be termed in a quantified way, as (r, α)-prox-regularity.

Key-words: Differential inclusions - Differential variational inequalities - Regularization - Proximal normal cone - Locally prox-regular set - Sweeping process - Perturbation.

Mathematics Subject Classifications (2010) : Primary 34A60 · 49J52 · Secondary 49J20 · 58E35

1 Introduction and related problems

The study of differential inclusions as

$$\begin{cases} \dot{x}(t) \in -N(C; x(t)) + F(t, x(t)) \text{ a.e. } t \in [0, T] \\ x(0) = x_0 \in C \end{cases} \qquad (D)$$

often appears in modelization in various fields. When C is a closed convex subset of a Hilbert space H, $F : [0, T] \times H \rightrightarrows H$ is a set-valued mapping, and $N(C, \cdot)$ denotes the usual normal cone, we may also write this differential inclusion in the variational form that for some $z(t) \in F(t, x(t))$ one has for almost every $t \in [0, T]$

$$\begin{cases} \langle -\dot{x}(t) + z(t), y - x(t) \rangle \leq 0 \quad \text{for all } y \in C \\ x(0) = x_0 \in C. \end{cases} \qquad (1.1)$$

C. Henry [18] introduced, with F independent of the time t, for the study of planning procedures in mathematical economy the differential inclusion

$$\begin{cases} -\dot{x}(t) \in P_{T_C(x(t))}(F(x(t))) \text{ a.e. } t \in [0, T] \\ x(0) = x_0 \in C, \end{cases} \qquad (1.2)$$

where $P_{T_C(x(t))}(F(x(t)))$ is the metric projection of the image $F(x(t))$ onto the tangent cone $T_C(x(t))$. In [18] the space H is finite dimensional, the closed set C is convex, and the set-valued mapping F is upper semicontinuous with nonempty compact convex values. In [12], B. Cornet reduced, as done in [18] under the convexity of C, the preceding problem to the existence of a solution of the differential inclusion

$$\begin{cases} -\dot{x}(t) \in N(C; x(t)) + F(x(t)) \text{ a.e. } t \in [0, T] \\ x(0) = x_0 \in C \end{cases} \qquad (1.3)$$

Dario Monticelli (Università degli Studi di Milano)

On fully nonlinear CR invariant equations on the Heisenberg group

In this talk we will provide a characterization of second order fully nonlinear CR invariant equations on the Heisenberg group. This result is the analogue in the setting of geometry on Cauchy-Riemann manifolds of the result proved in the Euclidean setting by A. Li and Y.Y. Li in [Comm. Pure Appl. Math., 56:1416–1464, 2003]. We also prove a comparison principle for solutions of second order fully nonlinear CR invariant equations defined on bounded domains of the Heisenberg group and a comparison principle for solutions of a family of second order fully nonlinear equations on a punctured ball.

Asymptotic stability of Landau solutions to Navier-Stokes system

Dominika Pilarczyk
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Joint work with **Grzegorz Karch**

It is known that the three dimensional Navier-Stokes system for an incompressible fluid in the whole space has a one parameter family of explicit stationary solutions, which are axisymmetric and homogeneous of degree -1. We show that these solutions are asymptotically stable under any L^2-perturbation.

The concentration-compactness principle for variable exponent spaces and applications

Analia Silva
Universidad de Buenos Aires

In this work join with Julián Fernández, we extend the well-known concentration – compactness principle of P.L. Lions to the variable exponent case. More precisely, we prove: Let $q(x)$ and $p(x)$ be continuous functions such that $q(x) \leq p(x)$ in Ω. Let $\{u_j\}$ be a weakly convergent sequence in $W_0^{1,p(x)}(\Omega)$ with weak limit u, and such that:

- $|\nabla u_j|^{p(x)} \rightharpoonup \mu$ weakly-* in the sense of measures.
- $|u_j|^{q(x)} \longrightarrow \nu$ weakly-* in the sense of measures.

Assume, moreover that $\mathcal{A} = \{x \in \Omega \colon q(x) = p^*(x)\}$ is nonempty. Then, for some countable index set I we have:

(0.1) $$\nu = |u|^{q(x)} + \sum_{i \in I} \nu_i \delta_{x_i} \quad \nu_i > 0$$

(0.2) $$\mu \geq |\nabla u|^{p(x)} + \sum_{i \in I} \mu_i \delta_{x_i} \quad \mu_i > 0$$

(0.3) $$S \nu_i^{1/p^*(x_i)} \leq \mu_i^{1/p(x_i)} \qquad \forall i \in I.$$

where $\{x_i\}_{i \in I} \subset \mathcal{A}$ and S is the best constant in the Gagliardo-Nirenberg-Sobolev inequality for variable exponents, namely

$$S := \inf_{\phi \in C_0^\infty(\mathbb{R}^n)} \frac{\| |\nabla \phi| \|_{L^{p(x)}(\mathbb{R}^n)}}{\|\phi\|_{L^{p^*(x)}(\mathbb{R}^n)}}.$$

We want to remark that in this Theorem is not required the exponent $q(x)$ to be critical *everywhere* and that the point masses are located in the *criticality set* $\mathcal{A} = \{x \in \Omega \colon q(x) = p^*(x)\}$. Moreover as an application of this Theorem , following the techniques of [1] we prove the existence of solutions for problems with lack of compactness.

References

[1] J. Garcia-Azorero and I. Peral, *Multiplicity of solutions for elliptic problems with critical exponent or with a nonsymmetric term.* Trans. Amer. Math. Soc. 323 (1991), no. 2, 877–895.

[2] P.L. Lions. *The concentration-compactness principle in the calculus of variations.* The limit case, part 1, Rev. Mat. Iberoamericana. Vol. 1 No.1 (1985), 145–201.

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Modelling the swelling of a cell due to osmosis.

Abstract:

The swelling of a cell due to osmosis can be modeled as a diffusion equation in two domains, separated by a free boundary moving by osmotic force and mean curvature. The resulting equation can be written as a gradient flow using Wasserstein metrics and geometric measure theory. Constructing a suitable space and metric poses some challenges: generally, at least one of the two phases will be non-convex, which results in the loss of some nice properties of the Wasserstein metric. Moreover, a degeneracy problem comes up when defining a metric which also incorporates movement of the boundary.

In this talk, the model will be introduced, and the main issues with defining the gradient flow will be presented.