



## Sixth JORNADES D'INTRODUCCIÓ ALS SISTEMES DINÀMICS I A LES EDP'S (JISD2007)

Barcelona, June 2007

The sixth edition of the JORNADES D'INTRODUCCIÓ ALS SISTEMES DINÀMICS I A LES EDP'S (**JISD2007**), will be held in Barcelona during June 2007, at the **Universitat Politècnica de Catalunya (UPC)**. This year the Jornades are organized jointly with the **Summer School Needs** (Nonlinear evolution equations and dynamical systems) and with **GLOBAL School on PDEs**: layers and dislocations.

The lecturing part of **JISD2007** and the **Summer School Needs** will be split in two different weeks. There will be three courses in the first week, four courses (of the Summer School Needs) in between, and two courses on the second week.

The courses belong to the Master in **Applied Mathematics**, inside the Graduate studies at UPC, and are organized by Prof. Xavier Cabré, Prof. Amadeu Delshams, and Prof. Tere M. Seara.

The **JISD2007**, as well as the Doctoral Programme, is supported by a Spanish grant of the "Ministerio de educación y ciencia": **Movilidad de profesorado Universitario en los programas de Doctorado que hayan obtenido la mención de calidad**. It is also supported by the grant Ayuda de movilidad asociada a los Masters oficiales (UPC), by the Proyecto **CONSOLIDER-MATEMATICA**, and by the **ESF** Programme GLOBAL.

There will be **full financial support** available for this edition, with priority to the students attending both **JISD2007** and the Summer School Needs.

Deadline to apply for financial support: 30 april 2007 (see Registration).

REGISTRATION FORM **fifth JISD'2007**

[You can see the courses' schedule here](#)

### Contents

1st week courses (June 11-15). **Will be held in in the room 007 of the FME building (Facultat de Matemàtiques i Estadística), at C/ Pau Gargallo, n. 5 Barcelona, 08028.**

Course	Abstract
<b>NORMAL FORM FOR HAMILTONIAN PDES</b> <b>Dario Bambusi</b> (Universita degli studi di Milano) <b>(Syllabus)</b>	Birkhoff's theorem for classical Hamiltonian systems. A model problem: the nonlinear wave equation. Birkhoff normal form for completely resonant PDEs, dynamical consequences: exponentially stable periodic solutions. Birkhoff normal form for nonresonant PDEs, dynamical consequences: approximatively invariant tori, almost global existence.
<b>INSTABILITIES IN THE 3 BODY PROBLEM</b> <b>Vadim Kaloshin</b> (Pennsylvania State Univ. and Univ. of Maryland) <b>(Syllabus)</b>	<ol style="list-style-type: none"> <li>1. Basic principles of classical mechanics: Newtonian mechanics, Lagrangian mechanics, Hamiltonian mechanics</li> <li>2. The N body problem: The 2 body problem, Collision and regularization, Final motions in the 3 body problem, Restricted 3 body problem, Ergodic theorems of classical mechanics.</li> </ol>
<b>INTRODUCTION TO THE PROPERTIES OF EXTENDED SYSTEMS</b> <b>Rafael de la Llave</b> (Univ. of Texas at	<ol style="list-style-type: none"> <li>1. Variational theory: We want to discuss some extensions of Aubry-Mather theory to systems on lattices and PDE's. Models considered: Frenkel-Kontorova Models in several dimensions. Elliptic equations. Ginzburg-Landau equations. Minimal surfaces.</li> <li>2. KAM theory. We want to study the existence of smooth quasi-periodic solitions. We will present versions of the KAM</li> </ol>

Austin)

**(Syllabus)**

theory in situations where there is a Lagrangian theory without any Hamiltonian counterpart.

3. Hyperbolic theory.
4. Reduction methods

Summer School Needs (June 16-17). **Will be held in the Hotel Ametlla Mar, in the village of L'Ametlla de Mar.**

**AN INTRODUCTION TO PATTERN FORMATION, *Alastair Rucklidge*** (Leeds University)

**PROPERTIES OF LOW DIMENSIONAL DYNAMICAL SYSTEMS IN THE LARGE, *Carles Simó*** (Univ. de Barcelona)

**THE TRANSITION FROM REGULAR TO IRREGULAR MOTION AS TRAVEL ON RIEMANN SURFACES, *Paolo M. Santini*** (Univ. di Roma "La Sapienza")

**SYNCHRONIZATION AND NETWORKS, *Steven H. Strogatz*** (Cornell University)

2nd week courses (June 18-22). **Will be held in the in the room S01 of the FME building (Facultat de Matemàtiques i Estadística), at C/ Pau Gargallo, n. 5 Barcelona, 08028.**

Course	Abstract
<p><b>MULTI PEAK SOLUTIONS AND SOLUTIONS OF SEMILINEAR EQUATIONS IN PHASE TRANSITIONS AND A CONJECTURE OF DE GIORGI</b></p> <p><i>Changfeng Gui</i> (Univ. of Connecticut)</p> <p><b>(Syllabus)</b></p>	<p>In this course, I will first introduce some of the fundamental methods in nonlinear partial differential equations such as variational methods, the maximum principle, etc.</p> <p>Then, I will discuss two types of interesting solutions: the spike-layer solutions which arise in the analysis of the shadow system of a biological pattern formation model (the Gierer-Meinhardt system); and the transition layer solutions which play an important role in the study of phase transition via the Allen-Cahn equation and its counterpart in system of equations.</p> <p>For the spike-layer solutions, I will emphasize the new variational methods for the existence of higher energy solutions with multiple concentration. Regarding transition layer solutions, I will talk about the symmetry of solutions in entire spaces including De Giorgi conjecture, the existence of triple, quadruple junction solutions, a new Hamiltonian type equality and its application, etc. The topics are related to current research interests.</p>
<p><b>INTRODUCTION TO DISLOCATIONS DYNAMICS</b></p> <p><i>Regis Monneau</i> (CERMICS-ENPC, France)</p> <p><b>(Syllabus)</b></p>	<p>We will give an introduction to dislocations dynamics. Dislocations are curves defects in a crystal. When a stress is applied on the crystal, these curves can move with a dynamics given by the normal velocity depending on the whole shape of the curves and on the interactions with the other defects in the crystal.</p> <p>Mathematically, this dynamics is described by non-local Hamilton-Jacobi equations in the framework of viscosity solutions.</p> <p>After presenting classical results on homogenization, we will give in particular some results about the homogenization of the dynamics of self-interacting dislocations. In the limit we recover an effective plastic law which involves a fractional Levy operator. This will be an opportunity to present the introduction of recent tools on homogenization.</p>

(\*) For further details, please contact Prof. Xavier Cabré (xavier.cabre@upc.edu), Prof. Amadeu Delshams (amadeu.delshams@upc.edu), and Prof. Tere M. Seara (tere.m-seara@upc.edu).





# "GLOBAL School on PDEs: layers and dislocations"

Barcelona, June 18-22, 2007

Universitat Politècnica de Catalunya

## PROGRAM:

- 2 courses of 10 hours each and 3 invited 1 hour talks

## COURSES: (running June 18-22. See ABSTRACTS below)

- **Changfeng Gui** (Univ. of Connecticut):  
"Multi peak solutions and solutions of semilinear equations in phase transitions and a conjecture of De Giorgi"
- **Regis Monneau** (CERMICS-ENPC, France):  
"Introduction to dislocations dynamics"

## INVITED 1-hour TALKS: (Sala d'Actes of the FME building)

- June 18 (16h - 17h) : **Ioannis Athanassopoulos** (University of Crete, Heraklion):  
"A free boundary problem of codimension two"
- June 18 (17h - 18h) : **Juan Luis Vázquez** (Universidad Autonoma de Madrid):  
"Stabilization with rates in fast diffusion and weighted Sobolev inequalities"
- June 19 (16h - 17h) : **Henrik Shahgholian** (Royal Institute of Technology, Stockholm):  
"The singular set of the free boundaries in two membrane problem"

## REGISTRATION, PRACTICAL INFO, and SCHEDULE:

- Students attending the [GLOBAL](#) School should register at the web page [JISD07](#).
- Note that there will be **financial support**, partially from GLOBAL, for students to attend the school.
- A REGISTRATION FORM (including financial support), PRACTICAL INFO, and the SCHEDULE for the GLOBAL school are posted in the [JISD07](#).
- The "GLOBAL School on PDEs" is part of a larger event: **JISD07**. Students and researchers attending the GLOBAL school may be interested in attending other courses and events taking place within [JISD07](#).
- "[GLOBAL](#)", [Global and Geometric Aspects of Nonlinear PDE](#), is a European Science Foundation (ESF) Programme.

For more info at this point, contact:  
xavier.cabre@upc.edu

## ABSTRACTS for the COURSES:

- **Changfeng Gui**  
"Multi peak solutions and solutions of semilinear equations in phase transitions and a conjecture of De Giorgi"

In this course, I will first introduce some of the fundamental methods in nonlinear partial differential equations such as variational methods, the maximum principle, etc.

Then, I will discuss two types of interesting solutions: the spike-layer solutions which arise in the analysis of the shadow system of a biological pattern formation model (the Gierer-Meinhardt system); and the transition layer solutions which play an important role in the study of phase transition via the Allen-Cahn equation and its counterpart in system of equations.

For the spike-layer solutions, I will emphasize the new variational methods for the existence of higher energy solutions with multiple concentration. Regarding transition layer solutions, I will talk about the symmetry of solutions in entire spaces including De Giorgi conjecture, the existence of triple, quadruple junction solutions, a new Hamiltonian type equality and its application, etc. The topics are related to current research interests.

- **Regis Monneau**

"Introduction to dislocations dynamics"

We will give an introduction to dislocations dynamics. Dislocations are curves defects in a crystal. When a stress is applied on the crystal, these curves can move with a dynamics given by the normal velocity depending on the whole shape of the curves and on the interactions with the other defects in the crystal.

Mathematically, this dynamics is described by non-local Hamilton-Jacobi equations in the framework of viscosity solutions.

After presenting classical results on homogenization, we will give in particular some results about the homogenization of the dynamics of self-interacting dislocations. In the limit we recover an effective plastic law which involves a fractional Levy operator. This will be an opportunity to present the introduction of recent tools on homogenization.



## JISD2007 SCHEDULE

<b>June 11</b>	09.00 - 11.00	V. Kaloshin / Lecture 1
	11.00 - 11.30	Cofee break
	11.30 - 13.30	R. de la Llave / Lecture 1
	15.00 - 18.00	D. Bambusi / Lecture 1 - 2

<b>June 18</b>	09.00 - 11.00	R. Monneau / Lecture 1
	11.00 - 11.30	Cofee break
	11.30 - 13.30	Ch. Gui / Lecture 1
	16.00 - 17.00	I. Athanasopoulos
	17.00 - 18.00	J.L. Vázquez

<b>June 12</b>	09.00 - 11.00	V. Kaloshin / Lecture 2
	11.00 - 11.30	Cofee break
	11.30 - 13.30	R. de la Llave / Lecture 2
	15.00 - 18.00	D. Bambusi / Lecture 3 - 4

<b>June 19</b>	09.00 - 11.00	R. Monneau / Lecture 2
	11.00 - 11.30	Cofee break
	11.30 - 13.30	Ch. Gui / Lecture 2
	16.00 - 17.00	H. Shahgholian

<b>June 13</b>	09.00 - 11.00	D. Bambusi / Lecture 4
	11.00 - 11.30	Cofee break
	11.30 - 13.30	D. Bambusi / Lecture 5
	15.00 - 17.00	R. de la Llave/ Lecture 3

<b>June 20</b>	09.00 - 11.00	R. Monneau / Lecture 3
	11.00 - 11.30	Cofee break
	11.30 - 13.30	Ch. Gui / Lecture 3
	15.00 - 17.00	

<b>June 14</b>	09.00 - 11.00	V. Kaloshin / Lecture 3
	11.00 - 11.30	Cofee break
	11.30 - 13.30	R. de la Llave / Lecture 4
	15.00 - 17.00	V. Kaloshin / Lecture 4

<b>June 21</b>	09.00 - 11.00	R. Monneau / Lecture 4
	11.00 - 11.30	Cofee break
	11.30 - 13.30	Ch. Gui / Lecture 4
	15.00 - 17.00	

	09.00 - 11.00	V. Kaloshin / Lecture 5
--	---------------	-------------------------

	09.00 - 11.00	R. Monneau / Lecture 5
--	---------------	------------------------

<b>June 15</b>	11.00 - 11.30	Cofee break
	11.30 - 13.30	R. de la Llave / Lecture 4
	15.00 - 17.00	

<b>June 22</b>	11.00 - 11.30	Cofee break
	11.30 - 13.30	Ch. Gui / Lecture 5
	15.00 - 17.00	

For more information, please contact Tere.M-Seara@upc.edu

*June 07 -RMC*



# JORNADES D'INTRODUCCIÓ ALS SISTEMES DINÀMICS I A LES EDP'S (JISD2007)

Barcelona, June 11-15 and 18-22, 2007

(June 11-15)

Course	Syllabus
<p><b>NORMAL FORM FOR HAMILTONIAN PDEs</b></p> <p><i>Dario Bambusi</i> (Universita degli studi di Milano)</p> <p><a href="#"><u>Schedule</u></a></p>	<ul style="list-style-type: none"> <li>• Lecture 1. (1 hour) Overview. <ul style="list-style-type: none"> <li>– the classical result by Birkhoff</li> <li>– a model problem (string equation).</li> <li>– the resonant case: construction of exponentially stable periodic solutions, metastable states in lattices</li> <li>– the nonresonant case: an abstract Birkhoff normal form theorem and its applications</li> </ul> </li> <li>• Lecture 2. (1-2 hours) Scheme of the proof of Birkhoff's theorem. Difficulties for the infinite dimensional extension: <ul style="list-style-type: none"> <li>(i) the existence of non smooth vector fields,</li> <li>(ii) approximate resonance of finite combinations of frequencies.</li> </ul> </li> <li>• Lecture 3. (2 hour) Solution of the problem (i) when there are no small denominators: normal form for completely resonant systems, application to <math>u_{tt} - u_{xx} = u^3</math> with Dirichlet boundary conditions.</li> <li>• Lecture 4. (1-2 hours) Use of the normal form for the construction of families of exponentially stable periodic solution. The problem of metastability in nonlinear lattices of coupled particles.</li> <li>• Lecture 5. (4 hours) The nonresonant case (solution of the problem (ii)) <ul style="list-style-type: none"> <li>– A suitable class of nonlinearities;</li> <li>– A quantitative nonresonant condition;</li> <li>– Birkhoff normal form for some PDEs: a statement and its consequences.</li> <li>– Birkhoff normal form for some PDEs: proof.</li> <li>– Application to the nonlinear wave equation.</li> </ul> </li> </ul>
<p><b>INSTABILITIES IN THE 3 BODY PROBLEM</b></p> <p><i>Vadim Kaloshin</i> (Pennsylvania State Univ. and Univ. of Maryland)</p> <p><a href="#"><u>Schedule</u></a></p>	<ul style="list-style-type: none"> <li>• Lect. 1. Motion of Mars, and far out planets. Two body problem. Kepler laws.</li> <li>• Lect. 2. Central Configuration (CC). Collinear CC. Value of <math>\lambda</math>. Equivalence of CC. Saari conjecture.</li> <li>• Lect. 3. Collisions: simultaneous, pairwise. Sundman regularization.</li> <li>• Lect. 4. Restricted Planar Circular 3 Body Problem. Lagrange points. Hill regions.</li> <li>• Lect. 5. Hill Problem. Ergodic theorems of celestial mechanics.</li> </ul>
<p><b>INTRODUCTION TO THE PROPERTIES OF EXTENDED SYSTEMS</b></p> <p><i>Rafael de la Llave</i> (Univ. of Texas at Austin)</p> <p><a href="#"><u>Schedule</u></a></p>	<ul style="list-style-type: none"> <li>• Lecture 1. Variational theory We want to discuss some extensions of Aubry-Mather theory to systems on lattices and PDE's. Models considered: Frenkel-Kontorova Models in several dimensions. Elliptic equations. Ginzburg-Landau equations Minimal surfaces. <ul style="list-style-type: none"> <li>◦ 1.1 Existence of well ordered minimizers</li> <li>◦ 1.2 Existence of other critical points</li> <li>◦ 1.3 The average energy and the Peierls-Nabarro barrier</li> <li>◦ 1.4 Multibump solutions</li> </ul> </li> <li>• Lecture 2. KAM theory We want to study the existence of smooth quasi-periodic solutions. We will present versions of the KAM theory in situations where there is a Lagrangian theory without any Hamiltonian counterpart. <ul style="list-style-type: none"> <li>◦ 2.1 Moser's KAM theory for PDE's</li> <li>◦ 2.2 Lagrangian proof of the KAM theorem</li> </ul> </li> <li>• Lecture 3. Hyperbolic theory 3.0 Coupled lattices of Anosov systems <ul style="list-style-type: none"> <li>◦ 3.1 Structural stability</li> <li>◦ 3.2 Some interesting invariant measures</li> <li>◦ 3.3 Linear response functions</li> </ul> </li> <li>• Lecture 4. Reduction methods</li> </ul>

- 4.1 Invariant manifold theorems
- 4.2 Elliptic equations in cylinder domains
- 4.3 Water wave equations
- 4.4 The Weinstein approach to breathers
- 4.5 Traveling waves and traveling breathers
- 4.6 Chaotic solutions of elliptic PDE's

(June 18-22)

Course	Syllabus
<p><b>MULTI PEAK SOLUTIONS AND SOLUTIONS OF SEMILINEAR EQUATIONS IN PHASE TRANSITIONS AND A CONJECTURE OF DE GIORGI</b></p> <p><i>Changfeng Gui</i> (Univ. of Connecticut)</p> <p><a href="#"><u>Schedule</u></a></p>	<ul style="list-style-type: none"> <li>• Lecture 1 (2 hours): Allen-Cahn Equation <ul style="list-style-type: none"> <li>◦ 1d solutions</li> <li>◦ gradient estimate</li> <li>◦ monotonicity formula</li> <li>◦ Gibbons Conjecture</li> <li>◦ De Giorgi conjecture</li> </ul> </li> <li>• Lecture 2 (2 hours ): Vector-valued Allen-Cahn equation <ul style="list-style-type: none"> <li>◦ 1d heteroclinic solutions</li> <li>◦ triple junction</li> </ul> </li> <li>• Lecture 3 (2 hours): Vector-valued Allen-Cahn equation II <ul style="list-style-type: none"> <li>◦ double transition layer solutions</li> <li>◦ quadruple junctions</li> </ul> </li> <li>• Lecture 4 ( 2 hours): Young's Law and saddle solutions <ul style="list-style-type: none"> <li>◦ Hamiltonian identity</li> <li>◦ Young's law for triple junctions</li> <li>◦ 2d saddle solutions</li> </ul> </li> <li>• Lecture 5 (2 hours): Spike layer solution <ul style="list-style-type: none"> <li>◦ solutions in the entire space</li> <li>◦ variational methods and localized energy</li> <li>◦ single spike solution and least energy solutions</li> <li>◦ multiple spike layer solutions and high energy solutions</li> </ul> </li> </ul>
<p><b>INTRODUCTION TO DISLOCATIONS DYNAMICS</b></p> <p><i>Regis Monneau</i> (CERMICS-ENPC, France)</p> <p><a href="#"><u>Schedule</u></a></p>	<ul style="list-style-type: none"> <li>• Lecture 1. Introduction to dislocation dynamics <ul style="list-style-type: none"> <li>◦ Description of dislocations</li> <li>◦ The dynamics as a coupled system elasticity/ Hamilton-Jacobi equations</li> <li>◦ Elimination of the stress field via a non-local Levy operator</li> <li>◦ Classical results for viscosity solutions and application to dislocation dynamics</li> </ul> </li> <li>• Lecture 2. Mean curvature motion (MCM) as a singular limit of dislocation dynamics <ul style="list-style-type: none"> <li>◦ Slepcev formulation in the monotone case</li> <li>◦ Proof of convergence to MCM</li> <li>◦ Variational nature of the MCM</li> </ul> </li> <li>• Lecture 3. Homogenization of dislocations dynamics and of particle systems <ul style="list-style-type: none"> <li>◦ Homogenization in a simplified case</li> <li>◦ Correctors and convergence for dislocations dynamics</li> <li>◦ Application to systems of particles and mechanical interpretation</li> </ul> </li> <li>• Lecture 4. Global solutions to semilinear equations and application to homogenization of fronts <ul style="list-style-type: none"> <li>◦ The sliding method</li> <li>◦ Construction of periodic-in-time correctors</li> <li>◦ Application to the homogenization of fronts</li> </ul> </li> <li>• Lecture 5. Curved fronts and classification of solutions <ul style="list-style-type: none"> <li>◦ Proof of the classification in 2D</li> <li>◦ Construction of a curved solution via a Bernouilli free boundary problem</li> <li>◦ Existence and asymptotics of a solution to the Bernouilli free boundary problem.</li> <li>◦ Relations with the De Giorgi conjecture</li> </ul> </li> </ul>

