Primer Congreso Hispano-Francés de Matemáticas
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Special session *Integrable Systems* (July 12-13)

The aim of the session is to promote further exchange of ideas between the members of the groups in the following topics:

- Classical integrable systems and isomorphisms
- Darboux integrability for the polynomial systems
- Continuous and discrete integrable systems on Lie grupoids
- Applications of the Kovalevskaya-Painlevé method and its generalizations
- Applications of the differential Galois theory
- Algebraic integrability

The organizers

Guy Casale[*] *Université Rennes 1*

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[*]The members of ANR project [Intégrabilité réelle et complexe en Mécanique Hamiltonienne](#)

The Speakers[*]

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On the existence of solutions to the KdV hierarchy, doubly periodic in one of the variables

Armando Treibich

Spectral curves associated to solutions of the Korteweg-de Vries equation, doubly periodic in the variable x, have already been considered in relation with the Calogero–Moser elliptic integrable system or with the existence of closed geodesics and billiards on quadrics. More generally, one can consider the infinite dimensional KdV hierarchy and look for algebro-geometric solutions, doubly periodic with respect to the d-th KdV flow. We'll give the general properties of the corresponding spectral curves, and prove their existence (of arbitrarily high genus).

Keywords: elliptic KdV solutions, integrable Calogero–Moser systems, spectral curves

[1]Faculte de Sciences Jean Perrin
 Université d'Artois
 Postal address: Rue Jean Souvraz, 62300 Lens, France

Entropy, integrability, perturbations

Jean-Pierre Marco

Let (M^{2n}, Ω) be a smooth symplectic manifold and consider a Hamiltonian function $H \in C^\infty(M, \mathbb{R})$. We say that H is *tame-integrable* when it admits a smooth moment map $F = (f_1, \ldots, f_n) : M \to \mathbb{R}^n$ such that for each $r \in \{0, \ldots, n\}$, the singular locus $\mathscr{S}_r = \{x \in M \mid \operatorname{rank} F = r\}$ is an embedded submanifold (possibly empty) of M, on which Ω induces a symplectic form.

It is known that when H is tame-integrable, the topological entropy of the Hamiltonian flow of H (restricted to any compact invariant subset of M) is zero. Particular cases of tame-integrability are those when the singularities of F are nondegenerate in Eliasson sense (or in Ito sense, which is slightly better). We will first give some variations on this results in the framework of *weak* topological entropy, that we define as the *polynomial* growth rate of the number of balls in the usual coverings, in the case when the topological entropy vanishes.

We then deal with *perturbations* of completely integrable systems in action-angle form, defined on the annulus $\mathbb{A}^n = \mathbb{T}^n \times \mathbb{R}^n$, endowed with its angle-action coordinates (θ, r), by the following expression

$$H(\theta, r) = h(r) + f(\theta, r)$$

where h and f are two analytic functions, with holomorphic continuation on a fixed complex domain $\mathcal{D}$, such that $\varepsilon = \|f\|_{C^0(\mathcal{D})}$ is small. For such systems, we will analyze the topological entropy of the restriction invariant compact subsets of $\mathbb{A}^n$, as a function of ε as well as of the location of the invariant set relatively to the resonant zones of the unperturbed system. The core of the study is the splitting of the invariant manifolds of those hyperbolic tori which appear near the resonances as the result of the breaking down of the unperturbed Lagrangian tori.

Darboux polynomials at Darboux points

Jean Moulin Ollagnier

Darboux's method of integrability relies on the possibility of finding sufficiently many irreducible "Darboux polynomials". At the singular points of the system (Darboux points), the consideration of the linear part of the derivation gives important informations (LL method).

As a example, we prove rather simply that the multidimensional Jouanolou system $d(x_i) = x_{i+1}^S, 1 \leq i \leq n, x_{n+1} \equiv x_1$ has no polynomial first integral for any number $n \geq 3$ of variables when $S \geq 4$. Henryk Żołądek gave a very nice but intricate proof for all $S \geq 2$ a few years ago.

Keywords: Darboux polynomials, Jouanolou system

[1]Laboratoire LIX, Ecole Polytechnique
 Postal address: F 91128 Palaiseau Cedex, France

On the Galois groups of the q-hypergeometric equations

Julien Roques

The Galois group of a given linear differential equation can be computed using various methods more or less algebraic. In the regular singular case, a possible way to compute these groups is to use the Schlesinger density theorem (the monodromy is Zariski-dense in the Galois group). An analogous -but in various aspects different- result is available in the case of q-difference equations (J. Sauloy). After some preliminaries about Galois theory for q-difference equations, we will explain how one can use this result in order to compute the Galois groups of the so-called basic hypergeometric equations. We will insist on the difference with the differential case. Last, we will make some comments on the Galois groups of the generalized hypergeometric equations of order three.

Keywords: differential Galois groups, q-difference equations

[1]Laboratoire E. Picard,
 Universite Paul Sabatier
 Postal address: 118 route de Narbonne, 31000 Toulouse, France

Lie systems and integrability conditions of differential equations

José F. Cariñena

The geometric theory of Lie systems [1, 2] will be used to establish integrability conditions for several systems of differential equations, in particular some Riccati equation and Ermakov systems. Many different integrability criteria in the literature will be analysed from this new perspective, and some applications will be given

Keywords: Lie–Scheffers Systems, Riccati equation, Ermakov systems

Mathematics Subject Classification 2000: 34A26

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[1]Department of Theoretical Physics
 University of Zaragoza
 Postal address 50.009 Zaragoza, Spain
 `jfc@unizar.es`

Mechanical systems on Lie algebroids and Lie groupoids

Eduardo Martínez

Continuous Lagrangian mechanical systems defined on Lie algebroids can be discretized as Discrete Lagrangian mechanical systems defined on Lie groupoids. Many of the geometrical and dynamical properties (symplecticity, preservation of invariants, etc.) of classical mechanical systems are also exhibited by such more general systems. I will review the most relevant results in this area.

Keywords: Lie algebroids, Lie groupoids, reduction

Mathematics Subject Classification 2000: 53D20, 58H05

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Departamento de Matemática Aplicada
Facultad de Ciencias, Universidad de Zaragoza,
50009 Zaragoza, SPAIN
`emf@unizar.es`

Integrability and algebraic curves: Darboux's method

In 1878 Darboux [4] showed how can be constructed an integrating factor (and first integrals) of planar polynomial differential systems possessing a sufficient number of invariant algebraic curves. The integrating factor can be obtained easily using the expressions of the invariant algebraic curves. Hence, the key point in the Darboux theory of integrability is the concept of the invariant algebraic curve. Poincaré, in 1891 he asked the following question: For a fixed vector field give an effective algorithm in order to calculate the maximum degree of its invariant algebraic curves [6]. Jouanolou in 1979 he proved that for a fixed vector field the maximum degree of the invariant (irreducibles) algebraic curves is bounded. The vector field or has a finite number of invariant algebraic curves or has a rational first integral (and so all the orbits of the system are contained in algebraic curves)[5]. In this talk first we present a recent version of Darboux's method. Next we present the so called 'inverse problems of the Darboux theory of integrability': For a given set of curves which are the vector fields having these curves invariants? [2]. And which are the vector fields having an integrating factor formed by these curves? We also sketch the concept of multiple invariant curves: Intuitively, when some invariant algebraic curves 'collapse' then appears exponential factors. Multiplicity and integrability has mainly studding in [3] but has been also treated in [1] and pointed out the relation between multiplicity and Darboux integrability.

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Higher order Painlevé equations

Andrew Pickering

Higher order analogues of the Painlevé equations, whether derived in isolation through a classification programme, or whether derived as whole hierarchies of differential equations, are a topic of some considerable current interest. Here we give an overview of our results obtained for such higher order Painlevé equations, for example on the derivation of Bäcklund transformations. We also give results for higher order discrete Painlevé equations.

Departamento de Matemática Aplicada
Universidad Rey Juan Carlos, Madrid

andrew.pickering@urjc.es

ANALYTIC NON INTEGRABILITY AND HOMOCLINIC STRUCTURES

by

Jacky Cresson

Since the proof by Jürgen Moser [**5**] of non existence of analytic first integrals for diffeomorphisms of the plane possessing a hyperbolic fixed point with a transverse homoclinic crossing, many efforts have been devoted to generalize his approach. The basic idea behind the Moser analytic non integrability theorem is that there exists a very complicated set (a Cantor set) on which the dynamics is conjugated to a Bernoulli shift and this must prevent the system to possess an analytic first integral. This idea goes back to Henri Poincaré [**6**] in his famous work on the three body problem:

> "Que l'on cherche à se représenter la figure formée par ces deux courbes et leurs intersections en nombre infini dont chacune correspond à une solution doublement asymptotique, ces intersections forment une sorte de treillis, de tissu, de reseau à mailles infiniment serrées; chacune de ces courbes ne doit jamais se recouper elle-même, mais elle doit se replier elle-même d'une manière très complexe pour venir recouper une infinité de fois toutes les mailles du réseau. On sera frappé de la complexité de cette figure, que je ne cherche même pas à tracer. Rien n'est plus propre à nous donner une idée de la complexité du problème des 3 corps et en général de tous les problèmes de la dynamique où il n'y a pas d'intégrale uniforme."

In this talk, we will present several advances on this topic which analytic non integrability for diffeomorphisms possessing a homoclinic structure, *i.e.* an invariant object (a fixed point, an invariant normally hyperbolic or partially hyperbolic torus, and more generally an arbitrary normally hyperbolic compact manifold) with stable and unstable manifolds which intersect transversally or

not. An application of these results for the Birkhoff conjecture on Billiards will be given.

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JACKY CRESSON, Université de Pau et des Pays de l'Adour, France.
 E-mail : `jacky.cresson@univ-pau.fr`

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